COMPETITION AND ZERO PROFIT:

A Big Mess

Introduction

The neoclassical theory of income distribution does not only aim at demonstrating that, under competitive conditions, the remuneration of factors of production is equal to their marginal productivity. Since the distribution of income which results must respect principles of justice, theorists of this school, such as J.B. Clark and Walras, try to show that no agent can receive an income that does not correspond to some productive contribution. In other words, it is necessary for them to prove that remuneration of factors according to their marginal productivity exhausts the whole product.

A difficulty appears here. Indeed, there is no guarantee that the theorem of exhaustion of the product is verified under all circumstances, whatever the form of returns to scale. This difficulty has given rise to an ancient and recurrent controversy among neoclassical authors, an essential debate in the case of John Bates Clark and Wicksteed, debate in which Walras, Edgeworth, Wicksell, Hicks and Samuelson have participated actively. In this paper we begin by recalling the essential elements of the debate, and then consider the solution proposed by Samuelson after his critic of Hicks' demonstration. This solution, based on the assumption of 'free entry' and on a distinction between short period and long period equilibrium, is the one that eventually prevailed, as one can see by consulting textbooks in microeconomics. We then show that Samuelson's solution is inappropriate, and that, when returns to scale are not constant, the theory of remuneration of factors according to marginal productivity is in contradiction with the allegation that the product is exhausted.

This leads us to the conclusion that it is necessary to choose between two alternatives :

- either the assumption of decreasing returns to scale, which carries with it the existence of profits that the theory of remuneration according to marginal productivity is unable to explain (this profit does not remunerate any contribution to production)
- or the assumption of constant returns to scale, in which case the product is exhausted by

remuneration according to the theory in question, but all microeconomic arguments based on the assumption of a U shaped cost curve are no longer guaranteed. Paradoxically, neoclassical theory is then forced to reason in a context of prices determined exclusively by supply side conditions (as do classical and marxist theories).

Distribution and justice according to J. B. Clark

Generally considered the founder of the neoclassical theory of income distribution [1], John Bates Clark, in the first pages of his book *The Distribution of Wealth* (1899), stresses the "supreme importance" for « practical men, and hence for students" of the question of distribution, his objective being to demonstrate that "*where natural laws have their way, the share of income that attaches to any productive function is gauged by the actual product of it.* In other words, free competition tends to give labor what labor creates, to capitalists what capital creates, and to *entrepreneurs* what the coordinating function creates" (p 3., Clark's italics).

In order to create a "just" society, where everyone is remunerated according to his contribution to the product, Clark is in favor of suppressing all obstacles impeding the action of these "natural laws" -which for him are a synonym of "free competition"-,. The issue of justice implicit in the theory of distribution makes Clark assert that if "natural laws" are not allowed to prevail, workers would have the impression of being robbed, and many "would become revolutionists", and with good reason, given that "every right-minded man should become a socialist" (p 4). As "the *right* of the present social system to exist at all depends on its honesty" (p 5), it is essential to find "a principle that humanity can approve and perpetuate" (p 7). Such a "principle" can be found in the "natural laws" that lead to the complete distribution of the product among production factors, under the condition that they are remunerated according to their marginal productivity ; such distribution would not only be "just", but also efficient.

The question we want to raise is : which factor is remunerated by the profit ? At the beginning of his book, Clark introduces profits as the remuneration of the coordinating work of the *entrepreneur* : "This purely coordinating work we shall call the *entrepreneur's* function, and the rewards for it we shall call profits" (p 8). In order to simplify, he supposes that there are

[1] Significantly, the *New Palgrave Dictionary of Economics* (1987) reproduces Clark's 1925 article from the "old "edition of the *Palgrave's Dictionary of Political Economy*.

only two factors of production : labor and capital. He only considers stationary states, where profit must be nil (because it is not the result of the "contribution" of a production factor). But he doesn't really prove that profits are nil. In fact, Clark's reasoning is only valid if we assume that production functions are *homogeneous of degree one* (they exhibit *constant returns to scale*) . This point is at the core of the debate over profits.

Constant returns to scale, hidden assumption of Clark's analysis

Let us revisit Clark's reasoning, but using more modern mathematical language. Hence, if we denote by $F(\cdot)$ the production function and if $(K°,L°)$ is the economy's total stock of capital and labor, remuneration according to marginal productivity implies that we have $F'_K(K°,L°) = r$ and $F'_L(K°,L°) = w$, r being the interest rate, w the wage rate (the product acting as the *numeraire*). If the product is totally exhausted by the remuneration of capital and labor or, to put it otherwise, if there is no "undistributed residue" or "residual " suggesting some other factor to be remunerated we have :

$$r \cdot K° + w \cdot L° = F(K°,L°).$$

From there, we can state that if the "contribution" of each factor is measured by its marginal productivity, then :

(1) $$K° \cdot F'_K(K°,L°) + L° \cdot F'_L(K°,L°) = F(K°,L°).$$

Equation (1) is *Euler's identity* at $(K°,L°)$: the product is exhausted when each factor is remunerated at its marginal productivity. If we assume that Euler's identity is verified for every bundle (K,L), and that $F(\cdot)$ is continually differentiable, then it follows from the reciprocal of Euler's theorem that $F(\cdot)$ is a homogeneous function of degree one. Furthermore, since in a « just » society "natural laws" must hold also at the level of each firm, their production functions will also be homogeneous of degree one[2].

It therefore follows from Clark's theory of distribution that production functions are necessarily, always and everywhere, homogeneous of degree one. A result that Wicksteed greets as "an analytic and synthetic law [...] which would hold equally in Robinson Crusoe's

[2] If the wage rate w and the interest rate r are globally ? fixed (they are the marginal productivities of $F(\cdot)$ in $(K°,L°)$), any firm -with a production function $f_i(\cdot)$ - will choose, in perfect competition, an input bundle (K,L) such that $(f_i)'_K(K,L) = r$ and $(f_i)'_L(K,L) = w$; since the exhaustion of product requires that :

$$K \cdot (f_i)'_K(K,L) + L \cdot (f_i)'_L(K,L) = f_i(K,L),$$

Euler's theorem holds (assuming that the formula above is true for any bundle (K,L)), and therefore $f_i(\cdot)$ is homogeneous of degree one .

island, in an American religious commune, in an Indian village ruled by custom, and in the competitive centers of the typical modern industries" (Wicksteed, 1894). A statement which provoked Edgeworth's comment : "There is a magnificence in this generalization which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage ; and rational conduct is a homogeneous function, adds the modern *savant*. A theory which points to conclusions so paradoxical ought surely to be enunciated with caution" (Edgeworth, 1904, p 31).

Edgeworth's critique

Edgeworth is opposed to the idea, expressed by Clark and Wicksteed, that the entrepreneur makes no profits. Indeed, Walras affirms, in *Eléments d'économie politique pure,* that the entrepreneur 'makes neither gain, nor loss'. According to Edgeworth, "no amount of authority and explanation can make it other than a strange use of language to describe a man who is making a large income, and striving to make it larger, as 'making neither gain, nor loss'" (p 25).

He also criticizes Barone, when this author tackles the question of profits and the entrepreneur ("it is the one obscure topic in Professor's Barone's brilliant studies in Distribution"). Barone starts by stating that "nothing else can be said but that profit is formed by the difference between the entire product and the remuneration of the various factors corresponding to their respective marginal productivity", later explaining that this is a temporary situation, since "with the increase in the number of the competing entrepreneurs the profit of the undertaking tends to lose more and more the character of residual claimant, and tends to conform to that of the law of marginal productivity" ; or, again, "in such conditions the law of marginal productivity extends to the remuneration of the entrepreneur ; and, after having remunerated all the factors (the work of the entrepreneur included) in proportion to their marginal productivity[...], there remains no undistributed residue" (Barone (1896), in Edgeworth, 1904, p 26-27).

When stated this way, the theory of income distribution based on marginal productivity would only be valid " as a limiting case", after an adjustment has occurred in the number of entrepreneurs. We thus return to the Clark-Wicksteed analysis, with an additional "factor of production", the work of the entrepreneur :[3] homogeneity of degree one is reestablished, by adding a new production factor.

[3]Remember (see footnote 1) that Clark evades the problem of the entrepreneur -and his remuneration- by supposing a steady state.

Obviously, Edgeworth does not subscribe to this view of things which comes down to reducing the entrepreneur to the status of a worker whose services are subject to supply and demand, eliminating precisely what makes him different. This is why, according to him, "it is only with respect to factors of production which are articles of exchange that the proposed law of remuneration, the 'law of marginal productivity' is fulfilled in a regime of competition" (Edgeworth, 1904, p 28). The quotation marks used when referring to the 'law of marginal productivity' speak for his skepticism concerning this alleged law. Edgeworth prefers to stick to common sense : since the entrepreneur exists, and he is "one of the parties to an exchange" (p 31), he must "normally" earn something. But this is not satisfactory for a theorist, specially since it concerns such sensitive question as income distribution.

Walras' hesitations

As in Clark, at the root of Walras' writings there is a normative issue : he tries to establish the conditions under which exchange is "just". This is the context in which it is affirmed that the entrepreneur makes no profit. Walras' argument differs from Clark's in (at least) one aspect : he assumes a constant coefficient production function, and, therefore, marginal productivity and returns to scale are also constant.

Walras nevertheless became acquainted with the theory of distribution according to marginal productivity, and this triggered an exchange of letters with Barone (Jaffé, 1965). Furthermore, he added as an appendix to his third edition of *Eléments d'économie politique pure* a "Note sur la réfutation de la Théorie anglaise du fermage de M. Wicksteed", where he proposes as a *post scriptum*, referring to Barone, a solution to the problem of distribution according to marginal productivity in the general case, where all production functions are not necessarily homogenous of degree one. Hicks, who ultimately agrees with Walras, says that his proof is very "crabbed and obscure", almost unintelligible. We will not give a detailed account of this proof (presented in "modern" jargon in footnote 6), but rather focus on the Walras conclusion :

"Therefore :

1. Free competition brings about the minimum price ;

2. Under this regime, the rate of remuneration of each production factor is equal to the partial derivative of the production function, namely the marginal productivity (...) ;

3. All of the product is distributed among the production factors (...)". (Walras, 1895. Walras' italics).

But the " Note " disappear from later editions of his *Eléments.* If the fourth edition contains the assertion that "all of the product is distributed among the production factors" (p 588), this formulation disappears in later editions as the statement that "the rate of remuneration of each factor is equal to the partial derivative of the production function" [4].

The Wicksell-Hicks solution

Hicks refuses to assume constant returns to scale, et want to show that it is not required in the demonstration of the theorem of exhaustion of the product. After having remarked that « if we persist in thinking of the factor which receives the residue as the 'entrepreneur ', we shall get into endless difficulties" (p 234), he proposes that we consider "our typical firm as a Joint Stock Company, and suppose the residue to fall to the capitalist as capitalist, management (so far as management is required) being hired like labor of other grades... Once we adopt this assumption, the most ordinary non-mathematical analysis shows that every factor must get its marginal product. For every *hired* factor must get its marginal product, since otherwise the demand for it would expand or contract ; and every *unhired* factor (which is 'acting as entrepreneur') must get its marginal product, since if it got more, some would be transferred from the hired to the unhired class"(p 234). Hence the conclusion that "this is a perfectly satisfactory line of argument, and it is evidently reasoning of this kind which has generally persuaded non mathematical economists (for example, J.B. Clark and his followers) that the 'adding-up' difficulty is a delusion. And we shall see that they are right " (p 235).

Hicks borrows his solution from Walras, which he considers as "altogether free from the objections to which Wicksteed's own solution is liable", but, while Walras, reasoning in terms of constant returns, only mentions the "minimum price", Wicksell points out that this price corresponds to an "optimal" scale of production since returns are first increasing and then decreasing.[5] He therefore writes, in the section on "The Theory of Production and Distribution" of his *Lectures of Political Economy* that "as a rule the best returns are obtained at some particular scale of operations for the firm in question ... this scale of operations is ... the 'optimum' towards which the firm must always, economically speaking, gravitate ; and as it

[4] For more details, see Rebeyrol, 1994, p.53-81.
[5] When the returns to scale are constant, the minimum price corresponds to any production scale.

lies at the point of transition from 'increasing' to 'diminishing returns' (relatively to the scale of production), the firm will here conform to the law of *constants* returns" (Wicksell, 1901-1906, p 129). This means that the unit cost curve is U-shaped, and that the "optimum" around which the firm "gravitates" is the point where the cost curve is at its minimum (and where, therefore, the profit is nil).

Since at that point the tangent to the curve is horizontal, Hicks' "solution" consists in assimilating or equating the unit curve cost to this tangent -since homogeneity of degree one implies that the unit cost curve is a horizontal line everywhere- and to use derivatives evaluated at that point. In other words, he adopts a *local* point of view. The product is exhausted but only "at the point of transition from 'increasing' to diminishing returns'".[6]

Samuelson's approach to the problem : long run equilibrium

In chapter IV of his *Foundations of Economic Analysis,* Paul A. Samuelson, discusses extensively the theory of income distribution, and show that two equilibrium conditions are required : "It has often been argued that not only must price (average revenue) under 'perfect' competition equal marginal cost, but also it must be equal to average cost so that the net revenue will be zero. This second condition has not always been recognized as being of an entirely different nature from the first. In this section an attempt will be made to distinguish between them. It is hoped that in so doing it will be possible to put the famous 'adding up' problem and homogeneity of the production function in its proper place" (Samuelson, 1947, p 84).

Actually, for Samuelson, there is no "proper place" for the homogeneity assumption of the production function ; he thinks that we can dispense with it altogether. He wants to preserve the marginal productivity theory of income distribution by avoiding the homogeneity assumption for the production function : « in reality, it is not on philosophical grounds that

[6] Hicks' proof follows from the first order condition verified the inputs bundle $(q_1^*,\dots, q_n^*)$ that minimises the unit cost $c(q_1,\dots, q_n) = (p_1q_1+ \dots + p_n q_n)/f(q_1,\dots, q_n)$, where $f(\cdot)$ is the production function of the firm. The first order condition, $c'_{qi}(q_1^*,\dots, q_n^*) = 0$, $i = 1,\dots,n$, can be written as :

$$(p_i f(q_1^*,\dots, q_n^*) - (\Sigma_j p_jq_j) f'_{qi}(q_1^*,\dots, q_n^*))/ f^2(q_1^*,\dots, q_n^*) = 0 \ , \ i = 1,\dots,n$$

From this :

$$p_i f(q_1^*,\dots, q_n^*) = (\Sigma_j p_jq_j) f'_{qi}(q_1^*,\dots, q_n^*) \ , \qquad i = 1,\dots,n.$$

Multiplying by q_i on both sides, and adding over i from 1 to n (after simplifying $\Sigma_j p_jq_j$) :

$$f(q_1^*,\dots, q_n^*) = \Sigma_i q_i^* f'_{qi}(q_1^*,\dots, q_n^*).$$

We find Euler's formula, but only at the point $(q_1^*,\dots, q_n^*)$ where the unit cost is minimum.

economists have wished to assume homogeneity, but rather because they were afraid that, if they did not do so, contradictions would emerge to vitiate the marginal productivity theory. This is simply a misconception as will be indicated below » (p 85).

The solution proposed by Samuelson consists in distinguishing "the conditions of equilibrium imposed from within the enterprise by its desire to maximize profits" from "conditions of equilibrium resulting from inter-competition among firms" (p 81-82). This is the distinction - more Marshallian than Walrasian - between short run and long run equilibrium. The transition from the first to the second is the result of "free entry", which shifts the demand curve « downwards » towards the long run equilibrium point - at which "total gross revenue equals total expenditure". Since this point -the minimum of the unit cost curve (which "under the conventional assumptions" is U-shaped)- is the same one referred to by Wicksell and Hicks, Samuelson arrives at the same conclusion (and thus saves the theory of marginal productivity). But he reminds us that this is a "theorem" deduced from the condition that total revenue equals total expenditure[7], and precisely "it is this last condition and the *forces* which lead to it that are of importance" (p 87, our italics).

Briefly : the marginal productivity theory of income distribution is saved by an appeal to "forces" so that in "the long run", and thanks to "free entry", profits vanish.

The integer problem and the inexistence of a long period equilibrium

Samuelson's solution, with the distinction between the short run (profit maximization by the firm) and the long run (unit cost minimization resulting from free entry), has definitely imposed itself. Nevertheless, the long run equilibrium -which results from the assumption of U-shaped cost curves and "free entry"- does not exist in the general case. In order to see why, we denote the minimum unit cost by c^* and the quantity offered by each firm at that price by q^* [8]. In order to have a long run equilibrium, it is necessary that the number n of firms "that enter" the market be such that :

$$nq^* = d(c^*) ,$$

[7] The result follows from replacing in the identity between income and expenditure : $p^*f(q_1,^* \ldots, q_n^*) = \Sigma i \; p_i q_i{}^*$ the price p_i of input i by $p^*f'_{qi}(q_1,^* \ldots, q_n^*)$ (consequence of the first order condition of profit maximisation : $pf'_{qi}(q_1, \ldots, q_n) = p_i$) . That is :

$$f(q_1^*, \ldots, q_n^*) = \Sigma_i q_i f'_{qi}(q_1,^*, \ldots, q_n^*) .$$

[8] We assume that all firms are identical, that is only the most efficient ones (those that produce q^* at the least cost) would be able to survive over the long run.

where $d(c^*)$ is the demand at price c^*. Hence, since $q^* > 0$ (consequence of U-shaped cost curves) :

(2) $$n = d(c^*)/q^*,$$

where n is an integer (since it indicates the number of firms), but $d(c^*)$ and q^* do not need to be (they are positive real numbers). In consequence, equation (2), typical of long run equilibrium, *is verified only exceptionally*. It is "the integer problem" (Mas-Colell and al., 1995, p339), which results from the fact that a long run equilibrium exists only when the demand curve intersects the supply curve in one of its "discontinuity points". This does not need to happen.

If there is no natural number verifying (2), a long run competitive equilibrium - in which the number of firms is endogenous- with zero profits does not exist. Thus, Wicksell, Hicks and Samuelson argue as if the economy were situated at a certain point - a given resource allocation -, but this point is not attainable. At the root of the problem is the fact that the supply curve is discontinuous -because the unit cost curve is supposed to be U-shaped, and therefore because there are decreasing and then increasing returns to scale.

This problem is noted by Mas-Colell and al., who explain : "It seems plausible, however, that when the efficient scale of a firm is small relative to the size of the market, this 'integer problem' should not be too much of a concern" (p 339). Note the prudence of these formulations : "it seems", "should not", "too much of a concern". To make their argument even more convincing, they appeal to intuition : "Intuitively, when the efficient scale is small, we will have many firms in the industry and the equilibrium, although not strictly competitive, will involve a price close to c^*". Finally, they insist on the fact that for this approximation to be valid (if we ignore the integer problem) the production unit must be "small" : "Thus, if the efficient scale is small relative to the size of the market [...], then ignoring the integer problem and treating firms as price takers gives approximately the correct answer" (p 339).
Tirole proposes also to sidesteps the difficulty by using the same kind of arguments : "next, we make the assumption that there are many potential firms [...]. Therefore, the assumption of free entry naturally leads to that of approximately zero profit (actually, this intuition is valid only if the market is sufficiently large). To simplify our calculations, we will often assume that profits are zero. This assumption may lead to the number of firms being calculated as a non integer. In such a case the actual solution, which must be an integer, is the integer closest to but not

exceeding the real number calculated" (Tirole, 1988, p 278).

A new assumption : the small efficient scale

Aware of the existence problem of long run competitive equilibrium, both Mas Colell and al. and Tirole suggest that the problem can be solved or mitigated if there are "many" firms, each one with a "small efficient scale".

But this new hypothesis that does not solve the problem, for two reasons :

In the strictly mathematical realm, the fact that there are "few" or "many" firms (and therefore few or many "points of discontinuity" below the demand curve) does not change the integer problem, even if the number of firms that enter the market tends to infinite, and therefore the small efficient scale tends to zero. Indeed, if we take as a measure of the set of real numbers the function that associates each interval to its length, then the measure of any set of points - each one of them being of nil length- is equal to zero. (see for instance Kirman (1982).

But the main question is if we can ignore this discontinuity, as we do for instance when we assume that the number of goods is a real number, even if we know that goods are not indefinitely divisible. If the discontinuity that appears in relation to the number of firms was of the same nature, we could simply not pay attention to it. But the discontinuity that we abstract from when we suppose that the quantity of goods is a real number is of an empirical nature, where here the discontinuity has a *theoretical reason* : it comes from the existence of an efficient scale arising from the U-shaped unit cost curve assumption. To ignore this discontinuity amounts to denying this assumption, and to reason as if the returns where constant, and this is precisely what the authors were trying to avoid.

Conclusion

Unless we assume constant returns to scale, profits, in the competitive equilibrium, are strictly positive. The assumption of free entry proposed by Samuelson does not solve the problem, since it can lead to the inexistence of the long period equilibrium. Two solutions are then possible :

- *either suppose that* the production functions are *all homogeneous of degree one* (Walras' solution) ; but, then, we have to avoid using U-shaped unit cost curves (replacing them by horizontal lines).
- *or accept the existence of positive profits*

This latter solution is the one adopted for instance by Debreu, in *Theory of Value*. The problem is that it does not answer the questions raised by Clark and Walras : The "residual" that J.B. Clark wanted to get rid of in the name of "justice", is here incorporated into the model, but never explained.

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